Reprinted from IFPress (2008) 11(2) 2,3

Conditional Ratio Statements

John M. Ennis and Daniel M. Ennis

Background: "Our product is 5 times more effective at reducing malodor than Brand X!" is a ratio statement with powerful consumer take-away. More generally, a ratio claim involves any statement meant to indicate that one product is superior to another by a multiplicative factor. The statement given above and a statement such as "Shown in studies to be 33% stronger than the leading brand!" are both examples of ratio claims. In a previous technical report we showed that ratios estimated from experiments have error associated with them and that this variation needs to be considered in order to avoid exaggerating a claim of superiority[1]. Typically ratio statements are used to compare improvements on some interval scale. Until recently all methods for producing meaningful ratio statements have assumed that these improvements are positive and no approach has allowed for the possibility that the products involved could have deleterious effects. In this report we revisit the topic of ratios and present a generalization of the existing methods.

Scenario: Your company has an interest in comparing the relative efficacy of its malodor treatment to that of a major competitor. You conduct an experiment using 48 panelists. Each panelist evaluates three chambers on a 7-point word-anchored scale with "1" labeled "no malodor present" and "7" labeled "extreme malodor present." The three chambers respectively contain malodor, malodor plus your product and malodor plus your competitor's product. The subjects are divided into 6 groups and the experiment is randomized so that each group evaluates the chambers in a unique order. Table 1 shows the results.

Chamber	"1"	"2"	"3"	"4"	"5"	"6"	"7"	Mean
Malodor	0	0	2	7	15	13	11	5.50
Malodor + Your Product	1	5	17	15	9	1	0	3.60
Malodor + Competitor	0	0	4	8	16	13	7	5.23

Table 1. Results of your experiment.

Your research supplier reports these results to you and recommends that you consider the ratio of the difference between the rating means of malodor only and the rating means of each product plus malodor. This ratio is (5.50-3.60)/(5.50-5.23), which is 7.04. This makes some sense to you but you're troubled that the ratings means are being used directly to make this comparison. You know that the malodor reduction required for someone to decrease a rating from a "3" to a "2" might be different from the reduction required for that person to decrease a "4" rating to a "3." Moreover, you are also concerned by the lack of any consideration for the variance in the data.

Extracting Ratio Information: As a first step to compare the malodor reduction properties of two products it is necessary to measure the products in such a way that a decrease in a single unit corresponds to the same reduction in malodor regardless of what the initial rating was. For instance the same amount of malodor reduction should occur when one product causes a drop from a "3" value on this scale to a "2" value as when another product causes a drop from a "5" value to a "4" value. In addition to finding a scale with these interval properties, it should also be the case that a score remaining constant means that no malodor reduction has occurred. In short malodor reduction should be measured on a ratio scale. Note that ratings data do not satisfy either of these ratio scale properties.

Following a Thurstonian approach let us imagine that each type of chamber used in your study is represented by a distribution on an interval scale for which smaller values mean less malodor. Since there many sources of variance associated with the perception of malodor, you can assume that the perceptual distributions corresponding to each item are normally distributed. Let δ_1 be the difference between the mean associated with the malodor and the mean associated with your product plus malodor, and let δ_2 be the difference between the mean associated with the malodor and the mean associated with your competitor's product plus malodor. If one assumes equal variance in these δ values then estimates of these δ values can be determined[2]. Since your examples involve large samples, you can also assume that these estimates, called *d′* values, are normally distributed. Note that *d′* values are differences of interval scale values and hence have ratio scale properties. The *d′* values for your ratings data can be obtained using IFPrograms™ and are listed along with their variances and covariance in Table 2.

Chamber	*d′* Value	Variance	Covariance
Malodor + Your Product	1.763	0.056	0.023
Malodor + Competitor	0.258	0.045	

Table 2. *d′* values, variances and covariance for malodor minus malodor plus treatment.

Supporting a Ratio Claim: Looking at the values in Table 2 it is tempting to form the ratio of *d′* values and state that your product is 6.77 times better than your competitor's at reducing malodor. Since there is variance in these estimates such a claim would be misleading. If you were to rerun your experiment you could very easily find a ratio of *d′* values that is much less than 6.77. A better approach is to use the variances in the *d′* values to estimate a lower bound on the ratio of product performance. You would then base your claim on the largest number for which repeated runs of the experiment yield a ratio of *d′* values at least as large as that number 95% of the time. Note that this probabilistic approach was advocated in our previous technical report on ratios and is mathematically equivalent under cerain conditions to the classical approach of Fieller in terms of estimating ratios[3,4,5,6].

Under many circumstances establishing a ratio claim in the manner described above would be straightforward. In your case however you have a problem because your competitor's product does not reduce malodor very well at all. Based on the results of your experiment it is conceivable that another run of the experiment would yield a negative value for the *d′* of your competitor's product. If this were to happen no positive ratio could accurately compare your product to your competitor's. In fact Figure 1 shows your competitor's product's poor performance could lead to a negative ratio of *d′* values more than 11% of the time. Ironically, without an additional statistical tool you might actually be penalized for having a weak competitor.

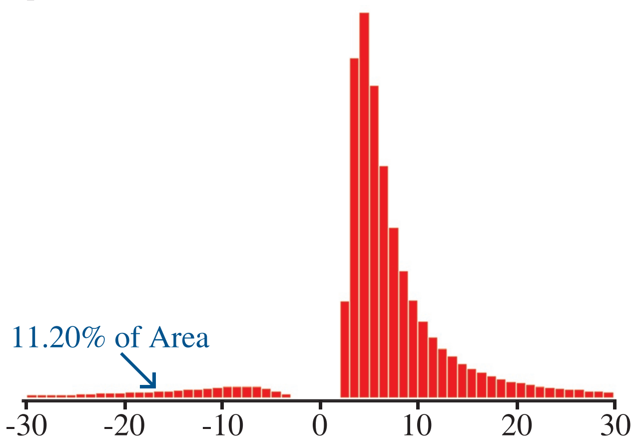

Figure 1. Results of Monte Carlo simulations showing the ratio of *d′*'s will be negative in 11% of the simulated runs of your experiment.

Conditional Ratio Statements: Recently Ennis et al.[7] extended the classical work on ratios to cases for which a competitor's weakness could produce a negative ratio. As a heuristic to understand this method you could again imagine running the original experiment over and over. This time however you would only consider runs of the experiment for which your competitor's product has a positive *d′*. Note that this approach is conservative since it allows your product to have either a positive or negative effect while only considering runs of the experiment for which your competitor has a positive average effect. Under these conditions you then find the largest number for which the ratio of *d′* values is at least as large as that number 95% of the time.

Results: Following this new method you find that the appropriate lower bound for your ratio is 3.04. This means that if you reran the original experiment until your competitor had a positive effect you would be 95% confident that the ratio of *d′* values would be greater than 3.04. This fact is illustrated in Figure 2. Based on this result you are now motivated to conduct a larger study to evaluate whether a claim that your product is 3 times more effective than your competitor's product at reducing malodor can be supported.

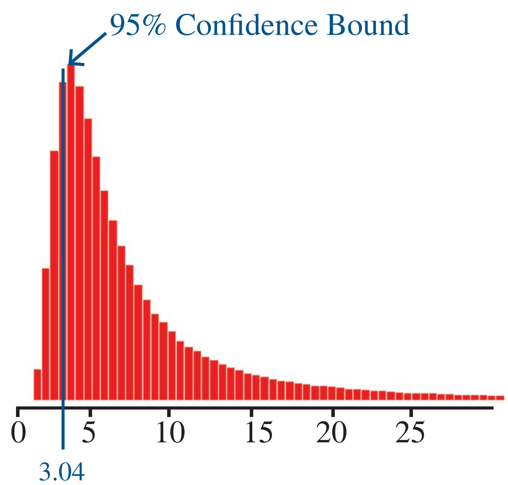

Figure 2. Results of Monte Carlo simulations showing 95% of the ratios of *d′*'s above 3.04 when simulated runs with negative competitor performance are excluded.

Conclusion: Once reliable interval scale data have been obtained, differences on interval scales can be used as terms in a ratio. Ratio claims can then be substantiated by a novel method that generalizes classical results such as Fieller's theorem and accommodates possible poor performance by a competitor.

References

[1]Ennis, D.M. (2006). Making Ratio Statements about Product Improvements. *IFPress*, **9**(2), 2-3.

[2]Thurstone, L.L. (1927). A law of comparative judgement. *Psychological Review*, **34**, 368-389.

[3]Fieller, E. C. (1932). The distribution of the index in a normal bivariate population. *Biometrika*. **24**, 428–440.

[4]Geary, R. C. (1930). The frequency distribution of the quotient of two normal variates. *J. Roy. Statist. Soc*. **93**, 442–446.

[5]Hinkley, D. V. (1969). On the ratio of two correlated normal random variables. *Biometrika*. **56**. 635–639.

[6]Marsaglia, G. (1965). Ratios of normal variables and ratios of sums of uniform variables. *J. Amer. Statist. Assoc.* **60**. 193–204.

[7]Ennis, D.M., Ennis, J.M., Palen, J. and Lampe, R. (2008). Confidence bounds for positive ratios of normal random variables. *Comm. in Stat.*, **37**. 307–317.