Reprinted from IFPress (2000) 3(1)2,3

Multivariate Sensory Quality Control

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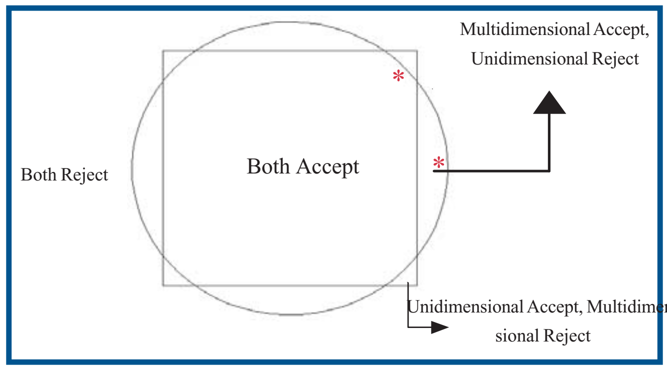
Background: A consumer product, such as a cookie, produces multidimensional sensory effects when it is eaten. In order to produce sensorially consistent cookies, it is necessary to develop and use methods that account for multidimensionality. For instance, a consumer may notice how dry or how hard a cookie is or how much moisture it absorbs when chewed. Some attributes may be independent of others; some may be interrelated. For example, we expect dryness and hardness to be related. In this report we show how to set multivariate specifications and how to establish control charts for sensory measures of food and beverage products. The ideas discussed have numerous applications in the analysis of multivariate attribute data.

Scenario: You have access to a database containing the rating responses of a trained panel with expertise in the texture evaluation of cookies. This panel has evaluated current production on a daily basis for the past year. The database contains information on fifteen sensory quality attributes of the current product from existing plants. The attributes used are those referred to by Meilgaard *et al.*[1] for texture evaluation, specifically: *roughness, loose particles, dryness, fracturability, hardness, particle size, denseness, uniformity of chew, moisture absorption, cohesiveness, tooth pack, grittiness, oily, particles,* and *chalky*. Your company has opened three new production facilities to produce the current product. Your objective is to decide whether the new plant produces cookies within the specifications established by the existing plant.

Limitations of Univariate Tests: A possible approach when evaluating multiple attributes is to consider them individually. In this case, a set of upper and lower control limits for each attribute is based on the mean and variance of each variable for the control set. In the cookie example, the control set is current production. Suppose, for example, that there are two independent attributes and that the data on these attributes have been standardized to normal z-scores. Hence these attributes have zero mean and unit variance. If 95% control limits are used on each attribute separately, then an acceptable product falls between -1.96 and +1.96 on each attribute. This leads to a square acceptance region for the two attributes. A variation on this approach is to set the Type I error at 5% based on the probability that *either* attribute is out of specification. In this case we obtain a square acceptance region extending from -2.24 to +2.24. This is because product is unacceptable if it falls outside *either* set of specifications with individual Type I errors of 0.025 (their sum is required to be 0.05.)

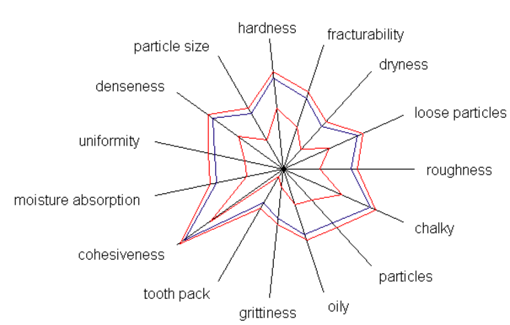
The distribution of two independent normal variables with equal variance such as the variables described above is circular. This means that points of equal likelihood are arranged on a circle around the mean. If we try to use a square acceptance region to assess values from a circular distribution, our conclusions will be inconsistent. For example, Figure 1 shows two points that are equally likely to occur. Nonetheless, the square acceptance region accepts one and rejects the other. Figure 1 illustrates the various cases that can arise with univariate 95% tests compared to a bivariate circular acceptance region with a radius of 2.447 (the 95% circular boundary for a standard bivariate normal distribution.) The discrepancy between the multivariate uniform tests and the single attribute tests may become even greater if attributes are correlated. In the general multivariate case, univariate tests involve a rectangular acceptance region, while the multivariate tests involve an ellipsoidal acceptance region of which the square and circle of Figure 1 are just special cases. Even in the most basic case, the univariate approach attempts to put a round peg into a square hole!

Figure 1. Acceptance/rejection regions for two attributes one at a time (square) and together (circle).



Limitations of Spider Plots: The spider plot has proven to be a useful tool to display sensory data. However, as a control chart, it has some limitations. Figure 2 shows the fifteen texture attributes for a cookie displayed in a spider plot with 95% confidence limits set on each attribute (the red lines.)

Figure 2. How a spider plot may erroneously display an out of specification product so that it appears to be acceptable. The red lines are upper and lower specifications, the blue line is the product.



A test product is displayed in this figure (the blue line) that falls within the upper and lower boundaries on each attribute. Nevertheless, this product is defective because it falls outside the 95% multivariate boundary for these attributes. Intuitively, it can be seen that this product has cumulative evidence that it is not typical of the control set, although we cannot find fault with it based on individual attribute analyses. The failure of Figure 2 to diagnose that the test product is outside the multivariate specification illustrates a weakness in using spider plots to display multivariate sensory data. Spider plots are really univariate displays of multivariate data and are not well suited to multivariate quality assessment of the type discussed.

Chi-square Control Charts: A product's sensory effects can be represented as a set of multivariate attributes. Assuming that this set is distributed multivariate normally, the attributes may be correlated or have different variances. We can transform the original attributes to a set of values that are distributed as a standard multivariate normal with mean zero, unit variance on all dimensions and zero correlation among dimensions[2,3]. The need for this transformation arises from the fact that sums of squared standard normal variables follow a chi-square distribution. If we transform a control set to standard form, we can consider whether some set of attributes, *y*, corresponding to a tested product, was drawn from this control set or not. In order to do this, we simply apply the same transformation to *y* and calculate the sum of squared elements of the resulting set. This value is a χ_n^2 with *n* degrees of freedom corresponding to the number of elements in *y*, which is the number of attributes. We can test this χ_n^2 using standard values.

Application: From the database on current production the means and variances for each attribute are computed along with the correlation coefficients among the attributes. The data are transformed to multivariate standard *z*-scores for which the mean is zero, variances are unity and attribute correlations are zero. Geometrically, we have transformed an ellipsoid to a sphere. Squared distances to the mean of the sphere are χ_n^2 distributed with *n* degrees of freedom equal to the number of attributes. The 95% confidence limit can be expressed in terms of χ_n^2. We find the value *k* so that $P\left(x_n^2 < k\right) = 0.95$. For a 15 variable problem, *k* is 25.0. For each of the plants, the data on each of the five days is transformed to multivariate *z*-scores using the transformation applied to the control set. Figures 3a, 3b, and 3c illustrate the chi-square control chart with data from the three new plants. The control limit corresponds to *k*. Notice that these control charts only have an upper limit because the chart displays squared distances to the mean specification. The first plant's data suggest that production is stable and within specification. The second plant's production is highly unstable. The third plant's production is stable but out of specification. Our recommendation is to accept the first plant's product and investigate processing conditions at plants 2 and 3 to find the cause of lower product quality.

Conclusion: Multiple, interdependent attributes are usually associated with consumer products. In order to measure and manage the quality of these products, multivariate techniques are important and useful. Individual attribute evaluations lead to two basic errors: Rejecting acceptable products and accepting rejectable products. Multivariate quality control tools manage these errors by appropriately accounting for multivariate effects and providing a sound theoretical basis for quality assurance. The reader is referred to our paper on multivariate quality control to learn technical detail regarding χ^2 control charts and other topics such as multivariate acceptance sampling[3].

Figure 3a. A plant with product meeting specifications.

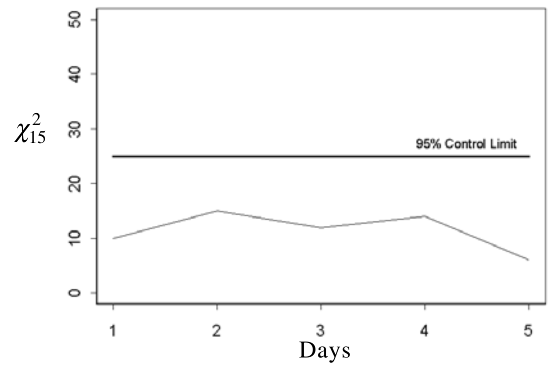


Figure 3b. A plant with highly variable product often out of specification.

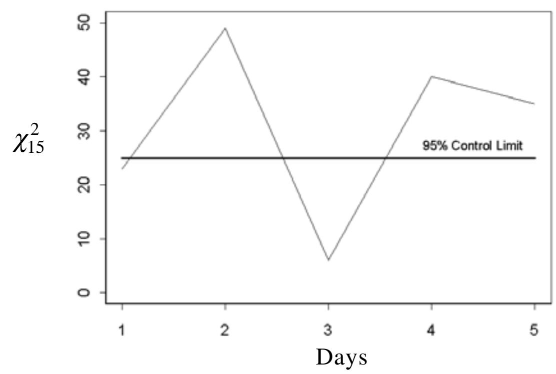
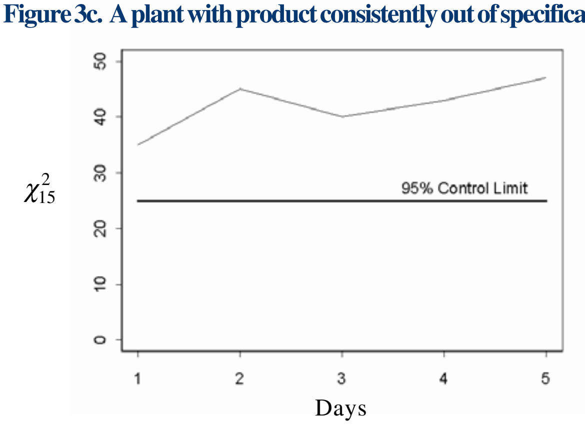


Figure 3c. A plant with product consistently out of specification.



References:

1. Meilgaard, M., Civille, G. V., and Carr, B. T. (1987). *Sensory Evaluation Techniques*. CRC Press: Boca Raton.
2. Ennis, D.M. (1993). Image inspection method and apparatus. *U.S. Patent* 5,208,870.
3. Ennis, D.M., and Bi, J. (2000). Multivariate quality control with applications to sensory data. Submitted.