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What is Science?

EuroSense
September 11, 2024

Representations or Models

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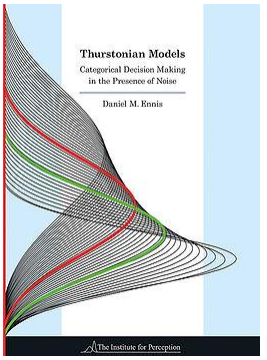
Books



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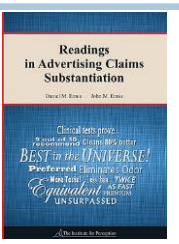

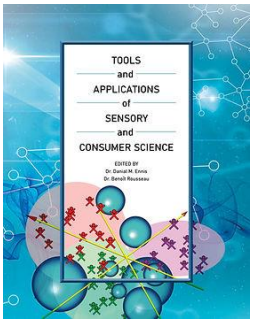

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1927: Louis L. Thurstone

- Percepts could be scaled using perceptual noise to create the scaling units and a decision rule
- Key to a theoretical foundation for methods in the behavioral sciences

1950: Clyde H. Coombs

- Liking model based on distance from ideal
- Degeneracies prevented implementation

2001: Daniel M. Ennis

- Novel Unfolding method based on a Thurstonian model of similarity, Landscape Segmentation Analysis® (LSA)
- Solved the degeneracy problem

Ennis, D. M., and Johnson, N. L. (1993). Thurstone-Shepard similarity models as special cases of moment generating functions. *Journal of Mathematical Psychology*, **37**(1), 104-110.

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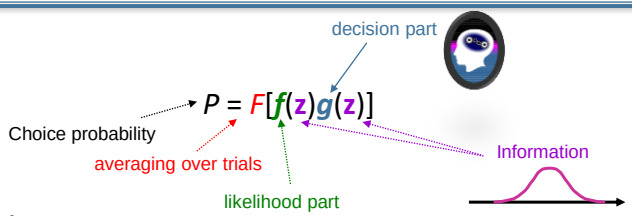
General Form of Thurstonian Probabilistic Models



- **Response (P)**
  - Weighted average of decision rule outcome
- **Examples**
  - Pc in discrimination tests
  - Same-different probability (Proportion of "same" answers)
  - Probability of a rating (Proportion of "7" on a 9-point scale)
  - Preference probability (Proportion of choice of Product A over Product B)

Thurstonian Models

2-AFC

$$P = \Phi\left(\frac{\delta}{\sqrt{2}}\right)$$

m-AFC

$$P = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp[-(z-\delta)^2/2]\,\Phi(z)^{m-1} dz$$

duo-trio

$$P = 1 - \Phi\left(\frac{\delta}{\sqrt{2}}\right) - \Phi\left(\frac{\delta}{\sqrt{6}}\right) + 2\Phi\left(\frac{\delta}{\sqrt{2}}\right)\Phi\left(\frac{\delta}{\sqrt{6}}\right)$$

Triangular

$$P = 2\int_0^{\infty}\left[\Phi\left(-z\sqrt{3}+\delta\sqrt{\frac{2}{3}}\right)+\Phi\left(-z\sqrt{3}-\delta\sqrt{\frac{2}{3}}\right)\right]d\Phi(z)$$

Dual Pair

$$P = 1 - G'\left(1;\frac{\delta^2}{2}\right)$$

G' is a singly non-central beta distribution

Multiple Dual Pair

$${}_{12}P_{34} = G''\left[\frac{\sigma_{34}^2}{\sigma_{12}^2};\left(\frac{\mu_{12}}{\sigma_{12}}\right)^2,\left(\frac{\mu_{34}}{\sigma_{34}}\right)^2\right]$$

G" is a doubly non-central beta distribution

Ratings 1 2 3 4 5 6 7 8 9

$$P_i = \Phi(b_i - \mu) - \Phi(b_{i-1} - \mu)$$

$$P = F[f(z)g(z)]$$

Tetrads

$$P = 1 - 2\int_{-\infty}^{\infty}\phi(x)[2\Phi(x)\Phi(x-\delta) - [\Phi(x-\delta)]^2]dx$$

Unspecified

$$P = 1 - 2\int_{-\infty}^{\infty}\phi(x)\Phi(x)[2\Phi(x-\delta) - [\Phi(x-\delta)]^2]dx$$

Specified

Preference, Triads, Identification

$${}_1P_{23} = \sum_{j=0}^{\infty}\sum_{g=0}^{\infty} e_{1j}\,e_{2g}\,\Pr\left[F_{n+2j,n+2g} < \frac{\beta_{2(n+2g)}}{\beta_{1(n+2j)}}\right]$$

F is a central *F* distribution

τ model

$$\sum_{j=0}^{\infty} e_j \Pr\left(\chi^2_{n+2j} < \frac{\tau^2}{\beta}\right)$$

or

Euclidean-Gaussian

$$(\|V\|J\|)^{-1/2}\exp[\mu'(2J^{-1}-I)\mu]$$

Same-Different

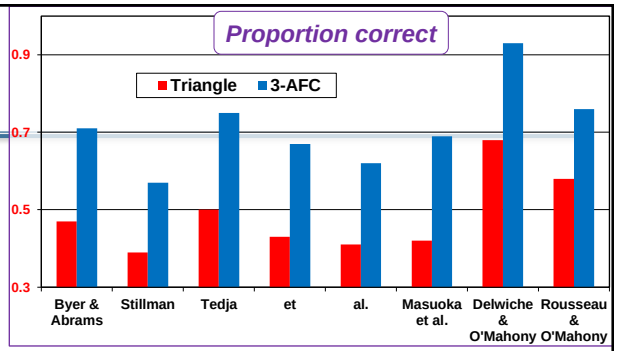
City-block-exponential decay

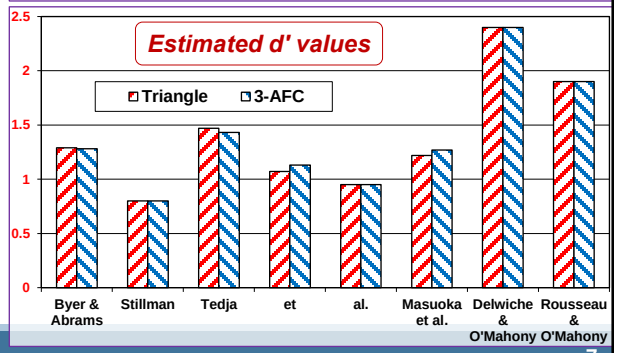
$$\prod_{i=1}^{n}\left(\exp[(2\mu_i+\sigma_i^2)/2]\Phi[-(\mu_i+\sigma_i^2)/\sigma_i] + \exp[-(2\mu_i-\sigma_i^2)/2]\Phi[(\mu_i-\sigma_i^2)/\sigma_i]\right)$$

Gridgeman's Paradox

Resolved by Jan Frijters (1979)

			# correct		d'	
Research	Product	N	Triangle	3-AFC	Triangle	3-AFC
Byer & Abrams, 1953	Bitter solutions	45	21	32	1.29	1.28
Stillman, 1993	Party onion dip	108	42	62	0.80	0.80
Tedja *et al.*, 1994	Salt Solutions	720 240 240	363 104 99	539 161 148	1.47 1.07 0.95	1.43 1.13 0.95
Masuoka *et al.*, 1995	Beer	108	50	75	1.22	1.27
Delwiche & O'Mahony, 1996	Choc. pudding	156	106	145	2.40	2.40
Rousseau & O'Mahony, 1997	Yogurt	180	105	152	1.90	1.90





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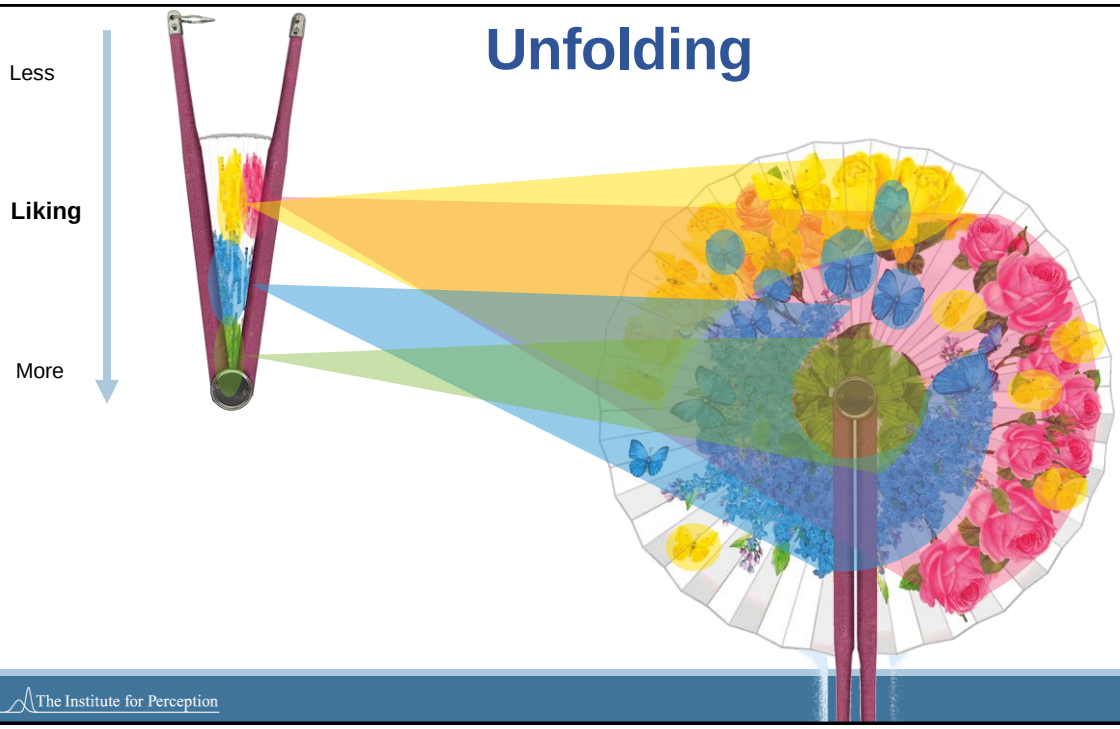
Unfolding



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IMAGINE IT IS 1972…

- High Interest in mapping techniques to understand consumer perceptions and choices as there is today
- Shepard developed Multidimensional Scaling (MDS) about a decade earlier
- The Coombs' ideal point to explain preference and liking is 2 decades old
- Difficulty implementing the Coombs model - Green and Rao get unexplained degeneracies
- Gabriel has just published the theory of biplots in 1971
- Carroll using Gabriel's theory publishes what is later known as Internal Preference Mapping (IPM) and External Preference Mapping (EPM)
- General agreement that the Coombs ideal point model is the most compelling but degeneracies block implementation
- Took until the early 2000s to solve degeneracy using a Thurstonian model of similarity
- 2001: Novel Unfolding method based on a Thurstonian model of similarity, Landscape Segmentation Analysis® (LSA), which solved the degeneracy problem

*Ennis, D. M., and Johnson, N. L. (1993). Thurstone-Shepard similarity models as special cases of moment generating functions. *Journal of Mathematical Psychology*, **37**(1), 104-110.

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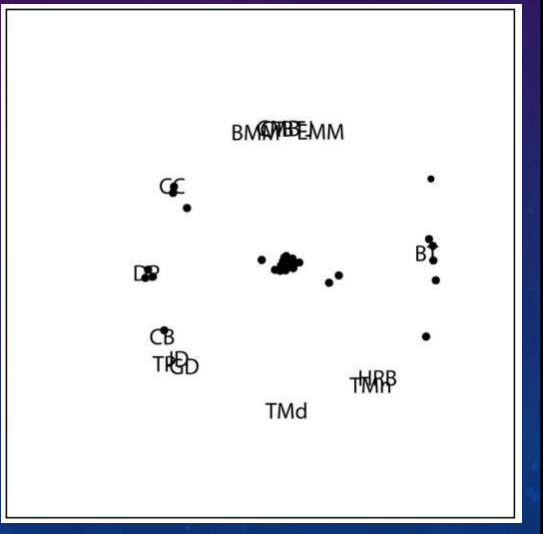
A DEGENERATE UNFOLDED SOLUTION
(GREEN AND RAO, 1972)

Code	Product	
BMM	Blueberry muffin and margarine	
BT	Buttered toast	
BTJ	Buttered toast and jelly	
CB	Cinnamon bun	
CC	Coffee cake	
CMB	Corn muffin and butter	
CT	Cinnamon toast	
DP	Danish pastry	

Code	Product	
EMM	English muffin and margarine	
GD	Glazed donut	
HRB	Hard rolls and butter	
JD	Jelly donut	
TMd	Toast and marmalade	
TMn	Toast and margarine	
TP	Toast pop-up	



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GREEN AND RAO REVISITED
UNFOLDING SOLUTION FROM LSA (2001)

Code	Product
BMM	Blueberry muffin and margarine
BT	Buttered toast
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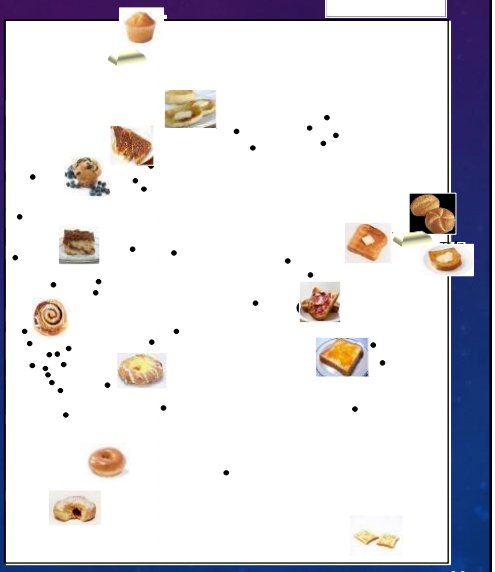

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Summary

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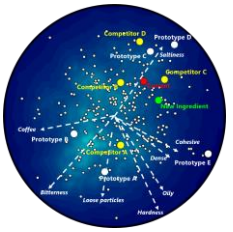

Science

- Developing process models
- Purpose of experimentation is to test and improve these models

The Thurstonian Theoretical Framework

- Models for an extremely wide range of methods
- Creates an opportunity for unification in the behavioral sciences

Categories of Applications

- Discrimination methods
- Category rating instruments from 2 to *m* alternatives
- Liking, preferential choice and similarity

Thurstone-Coombs

- Non-degenerate multidimensional mapping technique
- Identify individual ideal points and item distributions
- Enormous utility in new product innovation and brand development

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